Collective Diffusion and a Random Energy Landscape

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Abstract

Starting from a master equation in a quantum Hamiltonian form and a coupling to a heat bath we derive an evolution equation for a collective hopping process under the influence of a stochastic energy landscape. There results different equations in case of an arbitrary occupation number per lattice site or in a system under exclusion. Based on scaling arguments it will be demonstrated that both systems belong below the critical dimension d_c to the same universality class leading to anomalous diffusion in the long time limit. The dynamical exponent z can be calculated by an $\epsilon = d_c - d$ expansion. Above the critical dimension we discuss the differences in the diffusion constant for sufficient high temperatures. For a random potential we find a higher mobility for systems with exclusion.

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I. INTRODUCTION

Many systems behave on the phenomenological level essentially randomly and therefore other approaches for the theoretical treatment have to be employed. The randomness, resulting from stochastic forces or be intrinsic in the underlying microscopic theory, inevitably leads to the description of such systems in terms of probabilities and expectation values[1,2]. The time development of probability is usually found using a master equation. The past years have seen an exciting new development based on the observation[3] of the close relationship between the Markov generator of the master equation and a time evolution operator acting on a many-particle Fock space[4,5], for some recent reviews compare[6,7]. The new insight has led to a series of remarkable exact solutions for the stochastic dynamics of interacting particle systems, for a recent overview see[8]. Despite of exact results the mentioned method has been also fruitful in an approximative description of other models such as the facilitated kinetic Ising system as a candidate for glassy systems[9–11] or in branching and annihilation random walks[12]. Whereas the original paper[3], see also[13], are concerned with a mapping of the master equation to a representation in terms of second-quantized bosonic operators a great progress for exact solvable models had been achieved by mapping to spin-1/2 Pauli-operators[8]. This mapping to spin systems applies to processes where each lattice site can be occupied by only a finite number of particles. Physically, this restriction may be hard-core constraints or fast on-site annihilation processes. Obviously, such a mapping simulates the exclusion principle for classical lattice models with in a cellular automata.

In the present paper the Fock space description is applied for systems far from equilibrium which are coupled to a heat bath. In particular, we discuss the collective hopping process of a classical many body system, coupled to the mentioned heat bath, and under the influence of a random energy landscape realized by a stochastic activation energy. The particles making random walks have to overcome spatially distributed energy barriers. As the consequence the hopping process is accomplished by a competing force field which can give rise to anomalous diffusion. Further, the analysis should be different considering both cases, the bosonic and

the exclusive ones. In the first case the particles should find more rapidly the local energy minima however because of that their mobility could be reduced. As the consequence of the random walk where the particles have to overcome spatially distributed energy barriers, the resulting effective force field can give rise to anomalous diffusion. It is well known that one of the reasons for an anomalous diffusive behavior can be traced back to the influence of a stochastic force field below a critical dimension[14,15]. An alternative way of self induced anomalous diffusion had been discussed recently[16] introducing a feedback coupling between the diffusive particle and its local environment. Both, the disorder and the memory controlled feedback may lead to a subdiffusive behavior or to localization. The analytical approach[16] could be confirmed by simulations in one and two dimensions where at the critical dimension $d_c = 2$ logarithmic corrections in the mean square displacement had been found[17,18].

Here, we demonstrate that the Fock space approach leads in both cases, bosonic and under exclusion, may lead to anomalous diffusion. Within the long time limit and on a large spatial scale both systems belong to the same universality class.

II. QUANTUM APPROACH TO NONEQUILIBRIUM

The analysis is based on a master equation

$$\partial_t P(\vec{n}, t) = L' P(\vec{n}, t) \tag{1}$$

where $P(\vec{n}, t)$ is the probability that a certain configuration characterized by a state vector $\vec{n} = (n_1, n_2 \ldots n_N)$ is realized at time t. There are two special cases, eithe each lattice site is occupied by an arbitrary number of particles $n_i = 0, 1, 2 \ldots$ or as in a lattice gas $n_i = 0, 1$. Further, the occupation numbers n_i are considered as the eigenvalues of the particle number operator defined by creation operators $d_i^\dagger$ or by annihilation operators d_i. The problem is to formulate the dynamics in such a way that the possible realizations for the occupation numbers are taken into account explicitly. The situation in mind can be

analyzed in a seemingly compact form using the master equation in a quantum Hamilton formalism[3,4,2,5,19], for a recent reviews see[7,8]. The dynamics is determined completely by the form of the evolution operator L', specified below, and the commutation relations of the underlying operators $d_i^\dagger$ and d_i. Within that approach[3] the probability distribution $P(\vec{n},t)$ is related to a state vector $\mid F(t)\rangle$ in a Fock-space according to $P(\vec{n},t) = \langle \vec{n} \mid F(t)\rangle$. The basic vectors $\mid \vec{n}\rangle$ are composed of the operators $d_i^\dagger$ and d_i. Using the relation

$$\mid F(t)\rangle = \sum_{n_i} P(\vec{n},t) \mid \vec{n}\rangle \tag{2}$$

the master eq. (1) can be transformed into an equivalent one in a Fock-space

$$\partial_t \mid F(t)\rangle = L \mid F(t)\rangle \tag{3}$$

where the operator L' in (1) is mapped onto the operator $L = \sum \mid \vec{m}\rangle L'_{mn} \langle \vec{n} \mid$ in eq.(3). It should be emphasized that the procedure is up to now independent on the realization of the basic vectors. Originally, the method had been applied for the Bose case[3,4,13]. Recently, an extension to restricted occupation numbers (two discrete orientations) was proposed[2,5,19]. Further extensions to p–fold occupation numbers[11] as well as to models with kinetic constraints[21] and to systems with two heat bathes[22] are possible.

As shown by Doi[3] the average of an arbitrary physical quantity $B(\vec{n})$ can be calculated by the average of the corresponding operator $B(t)$

$$\langle B(t)\rangle = \sum_{n_i} P(\vec{n},t) B(\vec{n}) = \langle s \mid B \mid F(t)\rangle \tag{4}$$

with the state function $\langle s \mid = \sum \langle \vec{n} \mid$. The evolution equation for an operator $B(t)$ reads now

$$\partial_t \langle B\rangle = \langle s \mid [B(t), L] \mid F(t)\rangle \tag{5}$$

As the result of the procedure, all the dynamical equations governed by the classical problem are determined by the structure of the evolution operator L and the commutation rules of the operators.

III. COUPLING TO A HEAT BATH

The evolution operator for a collective hopping process is different for an arbitrary occupation number, denoted as Bose case, or an restricted occupation number, denoted as Fermi case. For the last system the operator L_f reads[9]

$$L_f = \mu \sum_{i,j} \left(d_i^\dagger d_j - (1 - n_i) n_j \right) \tag{6}$$

where μ is the hopping rate between adjacent sites i and j. The occupation number operator $n_i = d_i^\dagger d_i$ is related to the spin operator by the relation $S_i = 1 - 2n_i$ and the commutation rule is $[d_i, d_j] = \delta_{ij}(1 - 2n_i)$.

For the Bose case we get

$$L_b = \mu \sum_{i,j} \left(d_i^\dagger d_j - n_j \right) \tag{7}$$

where $d_i^\dagger$ and d_i fulfills the Bose commutation rules. A generalization to processes under the coupling to a heat bath with a fixed temperature T is discussed in[9]. As demonstrated in[9,23] the evolution operator has to be replaced by

$$L_f = \mu \sum_{i,j} \left[(1 - d_i d_j^\dagger) \exp(-\beta H/2) d_i^\dagger d_j \exp(\beta H/2) \right] \tag{8}$$

where the hopping rate μ defines a microscopic time scale; $\beta = T^{-1}$ is the inverse temperature of the heat bath and H is the Hamiltonian as a measure for the energy. A further generalization is realized by introducing different local heat bathes is discussed in[22]. In the bosonic case the generalization to finite temperatures leads to

$$L_b = \mu \sum_{i,j} \left[(1 - \delta_{ij}) \exp(-\beta H/2) d_i^\dagger d_j \exp(\beta H/2) \right] \tag{9}$$

Here we study the case that the Hamiltonian H in eqs.(8,9) is simply given by a stochastic energy landscape defined by the energy functional

$$H = \sum_i \varepsilon_i n_i \tag{10}$$

Whenever the energy is positive the empty site is energetically favored. Further, ε is assumed to be a stochastic local energy the distribution of which will be introduced below based on the continuous representation. In this manner, the model describes a collective hopping process where the jumping particles are subjected to a local random energy ε_i which supports or prevents the hopping process with a probability proportional to $\exp(\pm\varepsilon_i/2T)$. Taking into account the commutation rules we get in both cases

$$e^{-\beta H/2} d_i e^{\beta H/2} = d_i e^{\varepsilon_i/2T} \qquad e^{-\beta H/2'} d_i^\dagger e^{\beta H/2} = d_i^\dagger e^{-\varepsilon_i/2T} \tag{11}$$

Using eq.(5) and the algebraic properties of Pauli–operators, the evolution equation for the averaged density reads

$$\begin{aligned} \mu^{-1}\partial_t\langle n_r\rangle = \sum_{j(r)} & [\exp\left((\varepsilon_j-\varepsilon_r)/2T\right)\langle n_j\rangle - \exp\left((\varepsilon_r-\varepsilon_j)/2T\right)\langle n_r\rangle \\ & - 2\sinh(\frac{\varepsilon_j-\varepsilon_r}{2T})\langle n_r n_j\rangle \end{aligned} \tag{12}$$

In the Bose case the evolution equation is much simpler.

$$\mu^{-1}\partial_t\langle n_r\rangle = \sum_{j(r)} \left[\exp((\varepsilon_j-\varepsilon_r)/2T)\langle n_j\rangle - \exp((\varepsilon_r-\varepsilon_j)/2T)\langle n_r\rangle\right] \tag{13}$$

Both equations reflect the conservation of the particle number which will be more transparent in a continuum representation. In the special case of a constant energy $\varepsilon_r = \varepsilon_j$ it results the conventional diffusion equation in a discrete version. When the energy changes from site to site the nonlinear eq.(12) is the first step of a whole hierarchy of evolution equations. Assuming now smoothly changing energy ε_r and density n_r a gradient expansion is appropriate up to the order l^2 where l is the lattice size. To make the expansion invariant under the underlying rotational symmetry we have to use the following identity

$$\begin{aligned} \sum_{j(r)} \exp((\varepsilon_j-\varepsilon_r)/2T)\langle n_j\rangle = & \sum_{j(r)}\langle n_r\rangle \\ & + \exp(-\varepsilon_r/2T)\sum_{j(r)}\left[\exp(\varepsilon_j/2T)\langle n_j\rangle - \exp(\varepsilon_r/2T)\langle n_r\rangle\right] \end{aligned} \tag{14}$$

Such an expression reads in a continuous representation including terms of the order l^2

$$z n(\mathbf{r},t) + \exp(-\varepsilon(\mathbf{r})/2T)\nabla^2\left[(\exp(\varepsilon(\mathbf{r})/2T) n(\mathbf{r},t)\right]$$

with the averaged density $\langle n_r \rangle \equiv n(\mathbf{r},t)$; z is the number of nearest neighbors. After decoupling the nonlinear term in eq.(12) and performing the continuous limit the density $n(\mathbf{r},t)$ obeys the following nonlinear diffusion-like equation

$$\mu^{-1} l^{-2} \partial_t n = \nabla^2 n + n(1-n)\frac{\nabla^2 \varepsilon}{T} + (1-2n)\nabla n \cdot \nabla \varepsilon / T \tag{15}$$

In a system with exclusion the density couples in a nonlinear manner to the stochastic energy field $\varepsilon(\mathbf{r})$. Due to the exchange coupling of the evolution operator L in eq.(6) the resulting equation (15) is a conserving one where the current is given by

$$\mathbf{j}_f = -\nabla n - n(1-n)\frac{\nabla \varepsilon}{T} \tag{16}$$

In the Bose case we find after performing the continuous limit the density $n(\mathbf{r},t)$ obeys the following exact equation

$$\mu^{-1} l^{-2} \partial_t n = \nabla^2 n + \frac{1}{T}\nabla[n \nabla \varepsilon] \tag{17}$$

The conservation law is manifested in the current

$$\mathbf{j}_b = -\nabla n - n\frac{\nabla \varepsilon}{T} \tag{18}$$

The resulting equation is nothing else as the conventional diffusion equation under an additional drift term where the Einstein relation is automatically fulfilled. Remark that one can derive a similar equation when the system is coupled to two heat bathes with different temperatures. In that case one has to replace $\varepsilon(\mathbf{r})/T$ by $\frac{\nu}{T(\mathbf{r})}$ where ν is the chemical potential and $T(\mathbf{r})$ is the local temperature, see also[22]. In the Bose case eq.(17) depends on the density in a linear manner. It is of Fokker-Planck-type when the density $n(\mathbf{r},t)$ is considered as the single probability distribution to find a particle at site $\mathbf{r}$ at time t. Such an interpretation is always possible because we have not taken into account any interactions. Therefore, the particles are independent from each other and the concentration field behaves as the probability distribution of a single particle of this system. Different to the case of an arbitrary occupation the current $\mathbf{j}_f$ includes a term $n(1-n)$ which is characteristic for systems with exclusion. Due to the exclusion principle the systems reveals a kind of correlation

which leads even in the mean field limit to a nonlinear current. Following the discussion for the Bose case eq.(15) can be interpreted as a nonlinear Fokker-Planck-equation for a single particle. The nonlinearity reflects a feedback of a particle to itself due to the excluded volume effect.

It seems to be more appropriate to introduce the force vector $\mathbf{f}(\mathbf{r}) = -\nabla\varepsilon(\mathbf{r})$ the evolution equation in the Bose case reads now

$$\mu^{-1}l^{-2}\partial n(\mathbf{r},t) = \nabla^2 n - \frac{1}{T}\mathbf{f}\cdot\nabla n - \frac{1}{T}\nabla\cdot\mathbf{f}n \tag{19}$$

In the Fermi case the corresponding equation is

$$\mu^{-1}l^{-2}\partial n(\mathbf{r},t) = \nabla^2 n - \frac{1}{T}\mathbf{f}\cdot\nabla n(1-2n) - \frac{1}{T}n(1-n)\nabla\cdot\mathbf{f} \tag{20}$$

When the force field $\mathbf{f}(\mathbf{r})$ is a stochastic one the system offers anomalous diffusive behavior[14,15].

IV. SCALING

Now let us discuss both equations when the force field is an stochastic pure spatial dependent field, the correlator of which is given by

$$\overline{f_\alpha(\mathbf{r})f(\mathbf{r}')} = \phi_{\alpha\gamma}(\mathbf{r}-\mathbf{r}'), \quad \overline{f_\alpha(\mathbf{r})} = 0 \tag{21}$$

After averaging over the distribution function of the force field the system is homogeneous depending only on the difference of the spatial coordinates. The most general form of the function $\phi_{\alpha\gamma}$ is given in a Fourier representation by

$$\phi_{\alpha\gamma} = A(\vec{q})(\delta_{\alpha\gamma} - n_\alpha n_\gamma) + B(\vec{q})n_\alpha n_\gamma \quad \text{with} \quad n_\alpha = \frac{q_\alpha}{q} \tag{22}$$

Introducing dimensionless variables $x \to x\Lambda^{-1}, \quad t \to t\Lambda^{-z}$, where z is the dynamical critical exponent and further $n \to n\Lambda^d$ and according to eq.(22) for constant A and B $\mathbf{f} \to \mathbf{f}\Lambda^{d/2}$ we find the critical dimensionality $d_c = 2$. For $d \leq 2$ the term proportional to $\nabla(\mathbf{f}n)$ is relevant

whereas the additional term in case of exclusive motion $\propto n\mathbf{f}\nabla n$ is only relevant for $d < 2/3$. That means for the physical dimension $d \geq 1$ both models belong to the same universality class, where only $d \leq 2$ the disorder is relevant. Physically the result is obvious because in the long time limit and for a large spatial scale the Fermi system can be considered to consist of blocks with an increasing size. The larger such a block the more irrelevant is to distinguish both cases, arbitrary occupation and restricted occupation. In case of $d \leq 2$ the system reveals anomalous diffusive behavior as it had been demonstrated for a similar model not for the density $n(\mathbf{r}, t)$ but for the probability to P to find a particle at time t at the point $\mathbf{r}$. Making the same calculation we end up with the flow equations for the dimensionless coupling parameters $D = \mu l^2, \quad a = \frac{A}{D^2T^2}K_d, \quad b = \frac{B}{D^2T^2}K_d$, with $K_d(2\pi)^d$: the volume of the d-dimensional unit sphere and $\epsilon = 2 - d \quad \xi = \ln(\frac{\Lambda_0}{\Lambda}$

$$\begin{aligned}
\frac{\partial D}{\partial \xi} &= D\left[z - 2 + \frac{a(d-1)}{d} - \frac{b}{d}\right] \\
\frac{\partial a}{\partial \xi} &= a\left[\epsilon - a + \frac{b(d-1)}{d}\right] \\
\frac{\partial b}{\partial \xi} &= b\left[\epsilon - \frac{a}{d}\right]
\end{aligned} \tag{23}$$

In the same manner one can derive an equation for the mean square displacement $R = \Lambda^2 s(D, a, b)$ with $s = \langle \mathbf{r^2} \rangle$ The flow equation can be written as

$$2s = \frac{\partial s}{\partial D}\partial_\xi D + \frac{\partial s}{\partial a}\partial_\xi a + \frac{\partial s}{\partial b}\partial_\xi b \tag{24}$$

That equation leads to a scaling behavior of the mean square displacement in the vicinity of the fixed points of eqs.(23). In order to keep the diffusivity D fixed to its bare value the effective dynamical exponent $z(\xi)$ satisfies $z(\xi) = 2 + b(\xi)/d + a(\xi)(1-d)/d$. When the disorder is irrelevant the fixed points are $a^\star = b^\star = 0$ the exponent is $z = 2$. For the fixed point $a^\star = \varepsilon d, \quad b^\star = 0$ it results $z = 2 - \varepsilon$ and for $a^\star = b^\star = \epsilon d$ we find $z = 2 + O(\epsilon^2)$. These values are well known[14,15]. At the critical dimension $d_c = 2$ we proceed on the following manner. The observation time t is related to an initial time t_0 by

$$t = t_0 \exp(\int_0^\xi z(\xi')d\xi') \tag{25}$$

Using eqs.(23,24 we can fix the scaling parameter ξ according to eq.(25 to be

$$\xi \simeq \frac{1}{2}\ln(\frac{t}{t_0}) + \frac{1}{2}\ln(1 + \frac{a_0}{2}\frac{t}{t_0})$$

where a_0 is initial value for the parameter a. From eq.(24) we find the following behavior for the mean square displacement

$$\langle \mathbf{r}^2 \rangle = c_1 \frac{t}{t_0} + c_2 \frac{t}{t_0}\ln(\frac{t}{t_0}) \tag{26}$$

where c_1 and c_2 are two non-universal constants. As expected the system reveals logarithmic corrections at the critical dimension.

V. BEHAVIOR ABOVE THE CRITICAL DIMENSION

The thermalized version of the Fock space representation, see eqs.(8,9), leads in the limit $T \to \infty$ to conventional diffusion. In the high temperature limit the particles are able to overcome each barrier and as the consequence of the stochastic hopping process one finds diffusive behavior in the long time limit independently on the underlying statistics. When the temperature is finite there appears a competition between two processes resulting in a different behavior for both systems. Bose particles can easily find a minimum within the energy landscape defined by the stochastic force. Particles with exclusion have to search for a longer time and on a larger scale to reach an appropriate potential minimum. From here one would conclude to an enhanced diffusivity. On the other hand, the mobility of Bosons is eventually reduced because they find more rapid a stable minimum. Due to the established universality for low dimensions a variation of the behavior should be only observed above the critical dimension. In this regime conventional perturbation theory should be applicable. Let us therefore present lowest order corrections to the the diffusion parameter D. The effective diffusivity is defined by

$$D_{eff} = \left| \overline{\frac{\partial n^{-1}(\vec{q},\omega)}{\partial q^2}} \right|_{q=0,\omega=0} \tag{27}$$

As well as the Bose and the Fermi system lead in second order, proportional to $\frac{1}{T^2}$, to non-trivial corrections which are also manifested in the averaged density $\overline{n(\mathbf{r},t)}$ or the averaged correlation function $\overline{n(\mathbf{r},t)n(\mathbf{r}',t')}$. Indeed, the Fermi system offers additional terms for the density or the correlation function compared with the Bose case. However those terms does not contribute at zero wave vector and hence there are not relevant corrections to the divergent part of D_{eff} for $d \leq d_c$. Above d_c the behavior of the effective diffusion coefficient can be estimated using a perturbative approach around the homogeneous solution denoted by $\overline{n}$. We get

$$\begin{aligned} D^f_{eff} &= D^b_{eff} + \frac{(1-\overline{n})\overline{n}}{DT^2} I \\ \text{with} \quad I &= \frac{4K_d}{d} I_1[B - A(d-1)] \end{aligned} \tag{28}$$

I_1 can be expressed by a momentum integral which is always positive in the mesoscopic regime $\Lambda > l$. For $B - A(d-1) > 0$, realized for a pure potential field (B is the relevant variable, see eq.(22)), eq.(28) leads to

$$D^f_{eff} > D^b_{eff} \tag{29}$$

Remark that the correction to the bare diffusion coefficient D is of the order $(1-2\overline{n})^2$, that means for the half-filled case there are no corrections. That reasonable result should be also valid in a more refined approach.

Because the homogeneous solution is not necessary a stable one we can also estimate the behavior using linear stability analysis around the stationary solution denoted as $n_s(\mathbf{r})$. Let us introduce $n(\mathbf{r},t) = n_s(\mathbf{r}) + y(\mathbf{r},t)$ then the correction $y(\mathbf{r},t)$ fulfills in the Bose case the equation

$$\partial_t y = D\nabla^2 y + \frac{D}{T}\nabla(y\nabla\varepsilon_b) \quad \text{with} \quad \mathbf{f}(\mathbf{r}) = -\nabla\varepsilon_b(\mathbf{r}) \tag{30}$$

Here $\varepsilon_b(\mathbf{r}) = \varepsilon(\mathbf{r}) - v$ is the true stochastic potential introduced by eq.(10) and v plays the role of the chemical potential which regulates the occupation number. In case of the exclusion model the deviation from the stationary solution $y(\mathbf{r},t)$ satisfies the same equation however

one has to replace the potential in the Bose case, given in eq.(30), by another effective potential

$$\varepsilon_b(\mathbf{r}) \to \varepsilon_f(\mathbf{r}) = 2T \ln \left[\frac{\cosh\left(\frac{\varepsilon(\mathbf{r})-v}{2T}\right)}{\cosh\frac{v}{2T}} \right] \tag{31}$$

We have gauged the potentials so that for $\varepsilon_f(\mathbf{r}) = 0$ also $\varepsilon(\mathbf{r}) = 0$. The hopping particles under exclusion are subjected to the modified stochastic energy landscape given by ε_f. Expanding ε_f in terms of ε we find the relation

$$\varepsilon_f(\mathbf{r}) \simeq -\tanh(\frac{v}{2T})\varepsilon(\mathbf{r}) \tag{32}$$

From here it results

$$\overline{\varepsilon_f(\mathbf{r})\varepsilon_f(0)} \simeq \tanh^2(\frac{v}{2T}) \quad \overline{\varepsilon_b(\mathbf{r})\varepsilon_b(0)} \tag{33}$$

The effective correlator of the disorder in the Fermi system is drastically decreased in comparison to the Bose case. This result is compatible with the previous discussion leading to eqs.(28,29). In particular in the vicinity of half-filling (where the chemical potential v is zero) the influence of the disorder is very weak. This special case corresponds to vanishing linear term expanding eq.(31) according to powers of ε. In the leading order we obtain

$$\varepsilon_f(\mathbf{r}) \simeq \frac{\varepsilon^2(\mathbf{r})}{4T}$$

Different to the Bose case the effective stochastic potential ε_f, eq.(31), is always positive definite, that means all the deep negative minima of the original stochastic potential become maxima and therefore they are not more available in case of Fermi system. Obviously, they are already occupied and hence they are not accessible for particles.

VI. CONCLUSIONS

In the present paper the collective hopping process on a lattice is studied systematically when the particles are subjected to a random energetic landscape manifested by a stochastic

energy profile. In particular, we have taken into account both cases, each lattice site is only occupied by one particle or each site can absorb an arbitrary number of particles. Physically, one expects different behavior. Whereas in the situation under exclusion a particle should spend more time for searching an appropriate energy minimum within the stochastic energy the bosons tend to reduce their mobility because they remain for a longer time in the local minima. A further influence on the motion of the particles is given by the coupling to a heat bath which supports the tendency that the system equilibrates. Starting on a master equation in a second quantized form both cases can be easily realized in terms of Bose-operators or spin-1/2 Pauli-operators. The annihilation and creation process of particles leads in both cases to a density gradient characteristic for a random walk. Due to the additional coupling to a stochastic energy each particle can not follow that gradient simply but it has to overcome an energy barrier at its starting point and at its end point. There appears a conflicting situation that a particle follows the density gradient but the energy at the starting point is higher than at the end point. In this manner it will jump from an occupied to an empty site however under mobilizing a higher amount of energy (lower temperature). The other situation consists of the fact that a particle follows the density gradient and the energy barrier at the starting point is lower than at the end point (high temperature regime). In this case the hopping process is highly supported by the energy landscape whereas in the previous one the process is restricted. As the consequence anomalous diffusive behavior should be realized below the critical dimension.

In the paper we have demonstrated that the Bose-as well as the Fermi-system belong below the critical dimension to the same universality class within the long time limit and on a large spatial scale. For an increasing scale the system can be considered consisting of blocks with an increasing number of particles. Thus, the cases of restricted and unrestricted occupation number per lattice site should be irrelevant. Despite of the universality the density and the correlation function of both systems are different, in particular for an intermediate interval. In particular, we have discussed the situation above the critical dimension where the diffusion constant can offer different behavior in both cases.

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